Energy conservation and Jarzynski equality are incompatible for quantum work

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QTD2021, 7 Oct 2021

In collaboration with

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UNIVERSITET

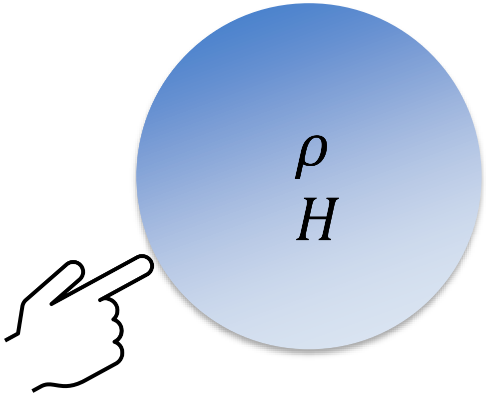
Plan of the talk

- Minimal requirements on work
- Linear measurement schemes
- Nonlinear measurement schemes
- The ultimate No-Go
- Summary

arXiv:2104.09364

Paradigm

Closed (⇔ thermally isolated) system is driven by external coherent fields



Main problem

– Can the fluctuations of work be observed?

– If we observe *some* statistics, how do we know if it is work?

Our approach

– Take the most general representation of work statistics quantum mechanics can offer

– Establish a minimal set of physical requirements work statistics must satisfy

– Check whether these requirements are consistent with the laws of quantum mechanics

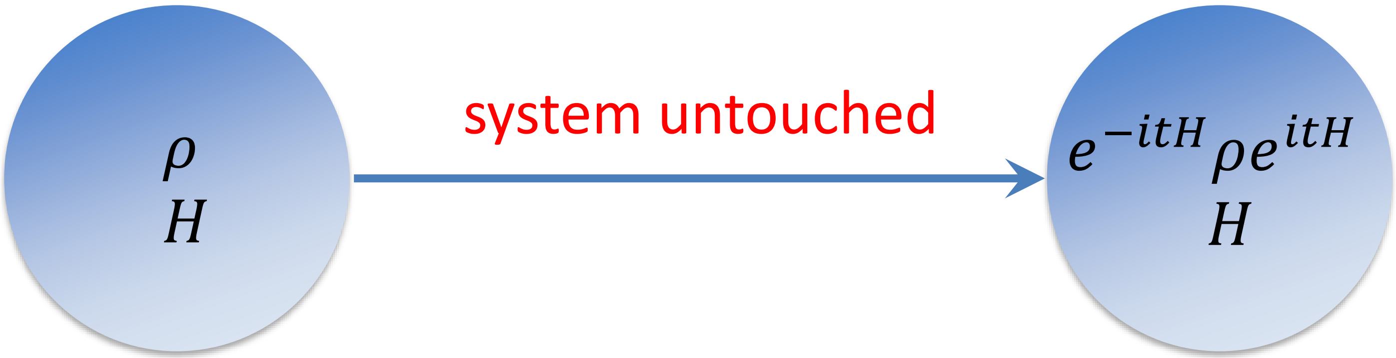
Inspired by the approach taken in

Perarnau-Llobet, Bäumer, Hovhannisyan, Huber, & Acin, PRL **118**, 070601 (2017)

- Minimal requirements on work

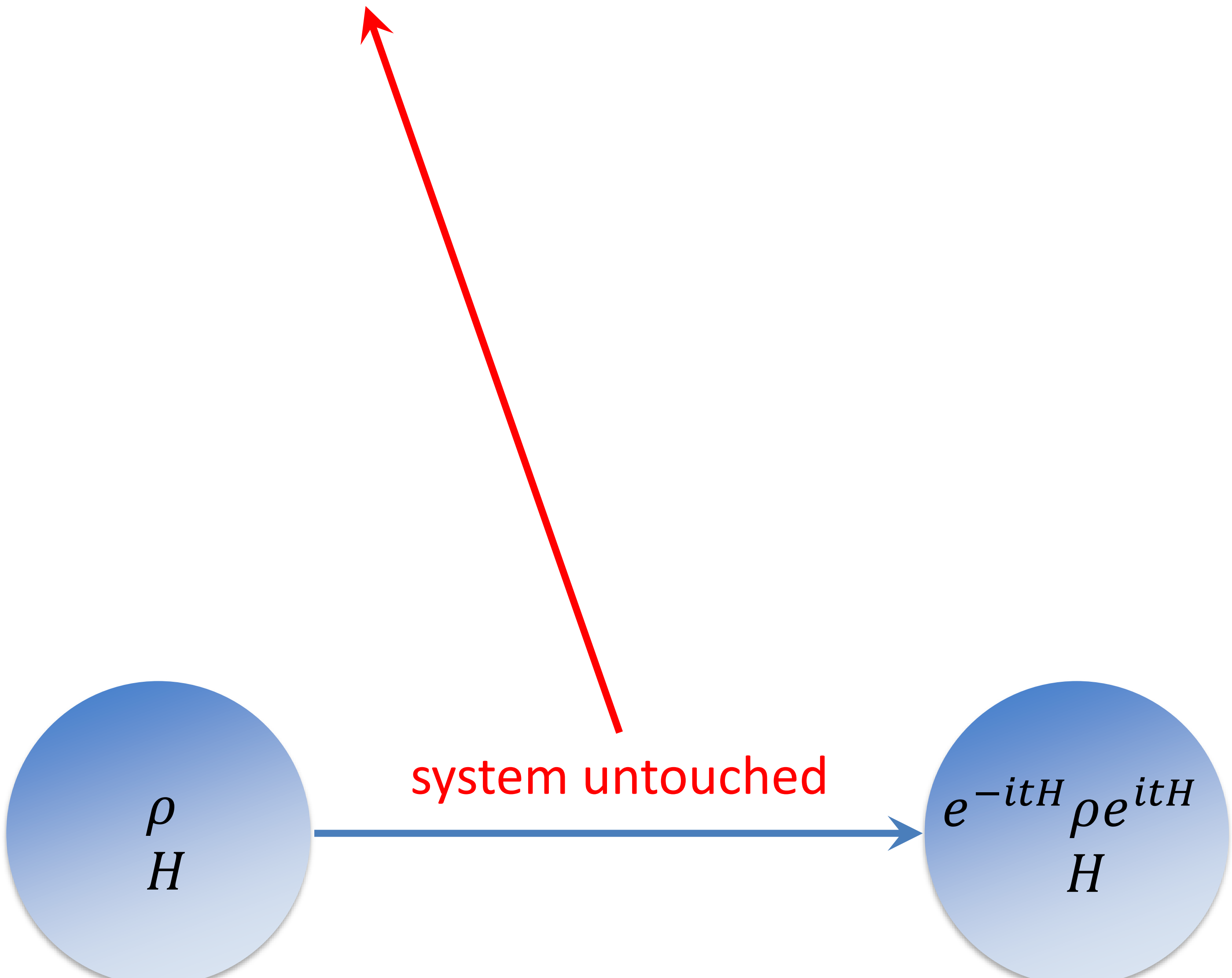
What is to be expected from work

(A) Nothing happens



What is to be expected from work

(A) Nothing happens $\Rightarrow$ deterministically zero work is exchanged



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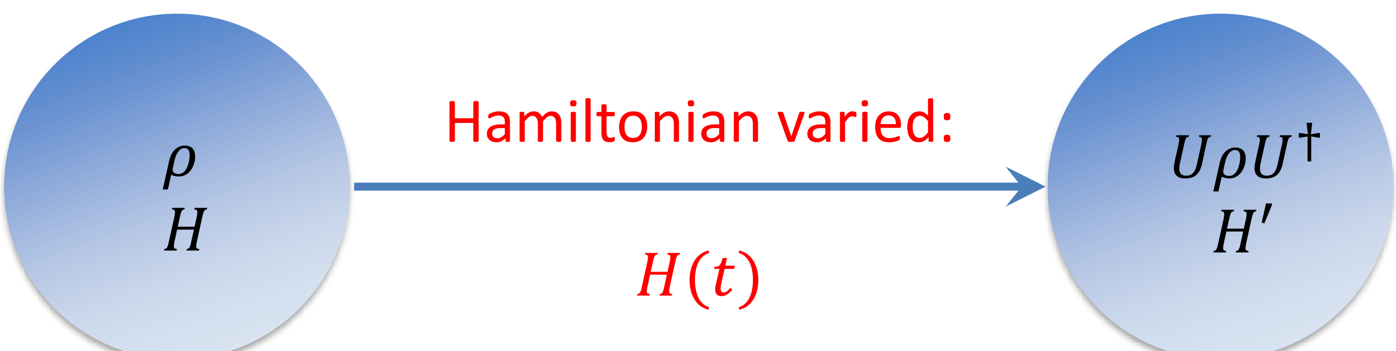
(A') System is being driven coherently (unitarily)



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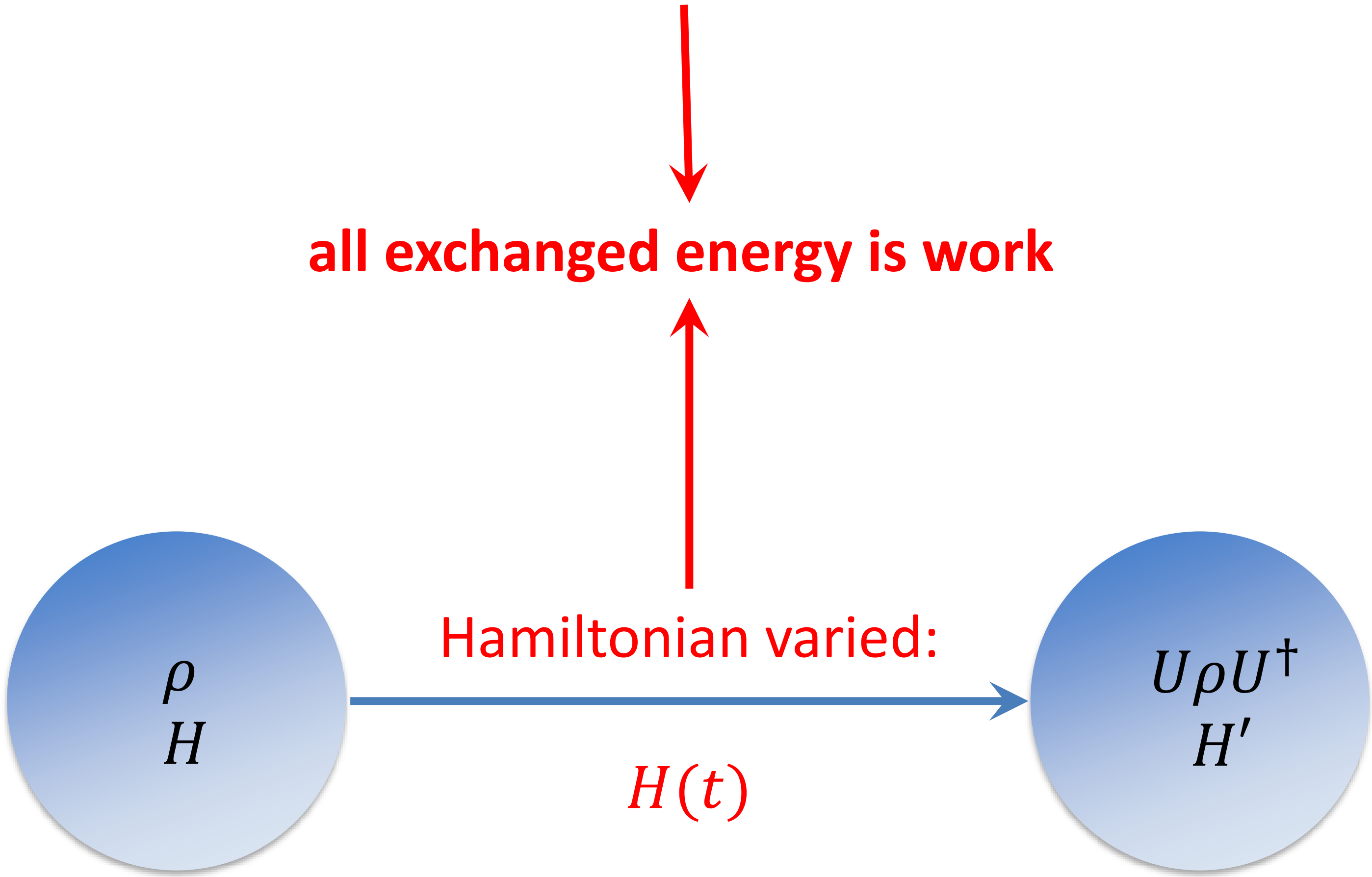

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$$\langle W \rangle = \mathrm{Tr}\left(U\rho U^{\dagger} H'\right) - \mathrm{Tr}(\rho H) \quad \forall \rho$$

both points ensure that the observed statistics respect the energy conservation

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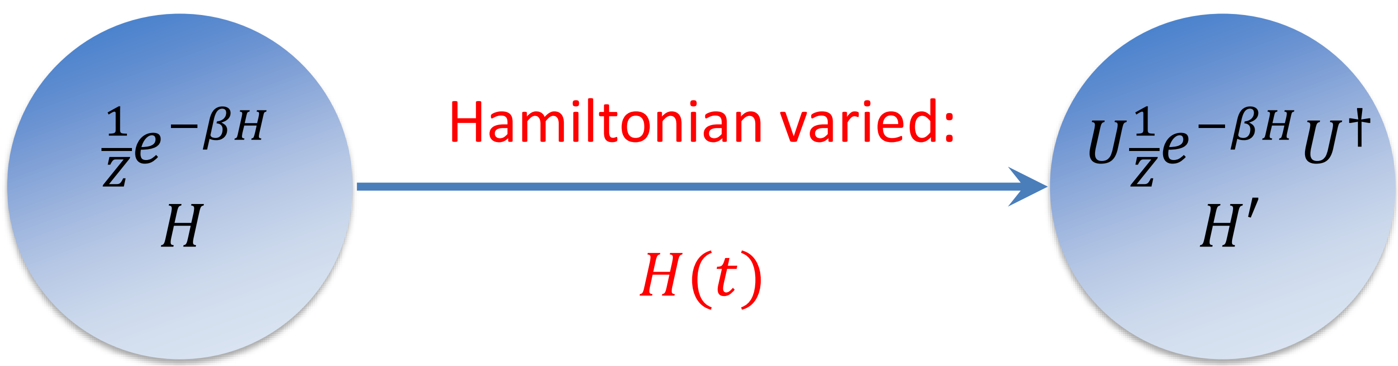
(B) Jarzynski equality holds when the system starts in a Gibbs state

$$\left\langle e^{-\beta W} \right\rangle = e^{-\beta \Delta F} \quad \forall \beta$$



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no a priori need for this, but it is a nice-to-have bridge to the second law and to the classical realm

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plus most major work-measuring protocols satisfy it

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Existing schemes and (A), (A'), (B)

Two-projective-measurement (TPM) scheme

Kurchan, arXiv:cond-mat/0007360

Tasaki, arXiv:cond-mat/0009244

Operator of work

Allahverdyan & Nieuwenhuizen, PRE **71**, 066102 (2005)

Bohmian mechanics approach

Sampaio, Suomela, Ala-Nissila, Anders, & Philbin, PRA **97**, 012131 (2018)

Bayesian network approach

Micadei, Landi, & Lutz, arXiv:2103.14570 [quant-ph]

“Single measurement” approaches

Deffner, Paz, & Zurek, PRE **94**, 010103(R) (2016)

Gherardini, Belenchia, Paternostro, & Trombettoni, arXiv:2106.06461 [quant-ph]

“Resource-theoretical” approach

Åberg, PRX **8**, 011019 (2018)

Alhambra, Masanes, Oppenheim, & Perry, PRX **6**, 041017 (2016)

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Miller & Anders, NJP **19**, 062001 (2017)

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Quasiprobabilities approach

for coherent processes, these papers do not construct a random variable for work, so they do not address the problem here

Allahverdyan, Phys. Rev. E 90, 032137 (2014)
Solinas & Gasparinetti, Phys. Rev. E 92, 042150 (2015)
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"Resource-theoretical" approach

Åberg, Phys. Rev. X 8, 011019 (2018)

while quasiprobability is a useful notion, constructing a quasiprobability distribution for work does not address the problem of measurability

Quasiprobabilistic approach

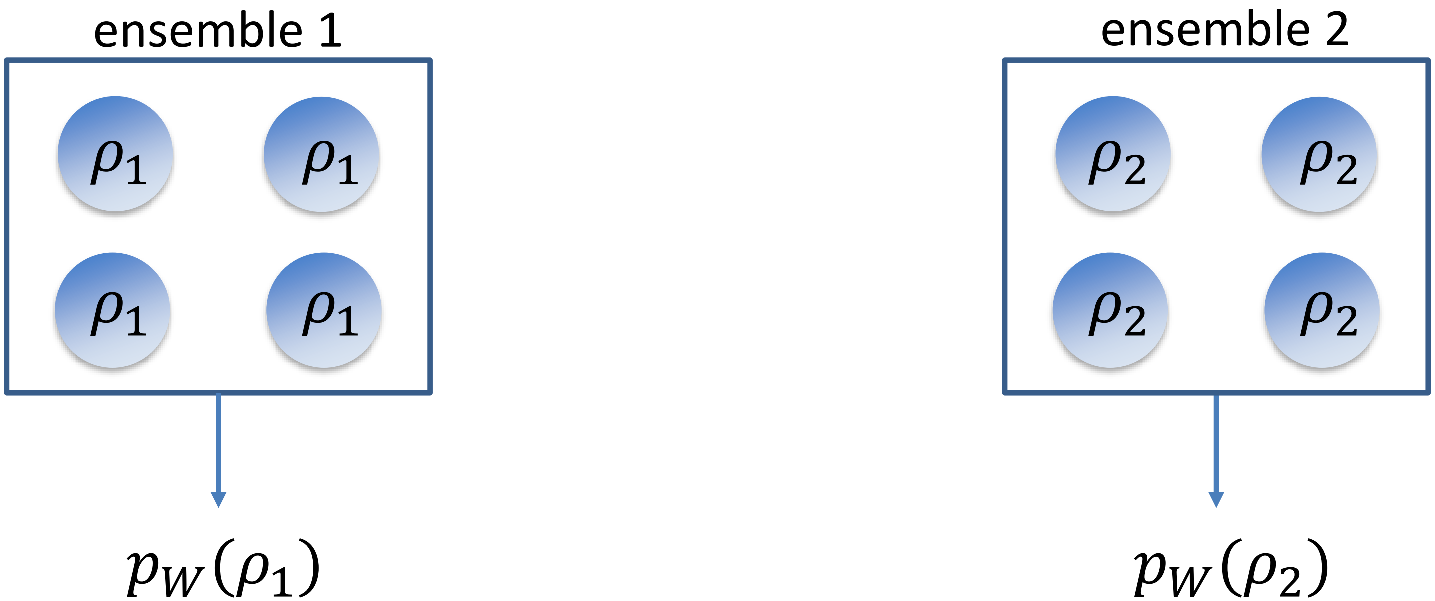
Allahverdyan, Phys. Rev. E 90, 032137 (2014)
Solinas & Gasparinetti, Phys. Rev. E 92, 042150 (2015)
Miller & Anders, New J. Phys. 19 062001 (2017)

The examples show that all the pairs of (A), (A'), and (B) are jointly satisfiable.

What about jointly satisfying all three?

- Linear schemes

Linearity



$$p_W(\lambda_1\rho_1 + \lambda_2\rho_2) = \lambda_1 p_W(\rho_1) + \lambda_2 p_W(\rho_2)$$

$$\forall \lambda_1, \lambda_2 \geq 0, \qquad \lambda_1 + \lambda_2 = 1$$

Linearity

$p_W(\rho_1)$ $p_W(\rho_2)$

$$p_W(\lambda_1\rho_1 + \lambda_2\rho_2) = \lambda_1 p_W(\rho_1) + \lambda_2 p_W(\rho_2)$$

$$\Updownarrow$$

$$p_W(\rho) = \mathrm{Tr}(\rho M_W)$$

constitute a POVM:

$$M_W \geq 0 \;\&\; \sum_W M_W = I$$

Perarnau-Llobet, Bäumer, Hovhannisyan, Huber, & Acin, PRL **118**, 070601 (2017)

Linearity

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M_W **are state-independent!**

Perarnau-Llobet, Bäumer, Hovhannisyan, Huber, & Acin, PRL **118**, 070601 (2017)

Requiring Jarzynski

Result 1

If a linear scheme satisfies (A') and (B) for all thermal states:

$$\sum_W W\ \mathrm{Tr}\left(\tau_\beta M_W\right) = \mathrm{Tr}\left(\tau_\beta U^\dagger H' U\right) - \mathrm{Tr}(\tau_\beta H)$$

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then it coincides with the TPM scheme:

$$M_W = M_W^{\mathrm{TPM}}$$

Jarzynski and TPM

TPM ⟹ **Jarzynski equality**

known from day 1

Jarzynski and TPM

our
Result 1

TPM ⇔ Jarzynski equality

known from
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Jarzynski and TPM

our Result 1

TPM ⇔ **Jarzynski equality**

known from day 1

Thus, linear schemes that satisfy (B) for thermal states cannot satisfy (A') for all states.

- Nonlinear schemes

Nonlinear schemes

Any conceivable distribution for work can be thought of as one generated by a POVM performed on the system:

$$p_W = \mathrm{Tr}(\rho M_W)$$

In general, p_W will **not** be linear in ρ:

$$M_W = M_W(\rho)$$

Nonlinear schemes

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Can this extra freedom allow reconciliating (A), (A'), and (B)?

Artificial nonlinear protocol

Artificial Protocol

- When $[\rho, H] = 0$, perform TPM

- When $[\rho, H] \neq 0$, perform the operator of work scheme

Artificial nonlinear protocol

Artificial Protocol

- When $[\rho, H] = 0$, perform TPM

- When $[\rho, H] \neq 0$, perform the operator of work scheme

$\Downarrow$

Conditions (A), (A'), and (B) are satisfied simultaneously!

Artificial nonlinear protocol

Artificial Protocol

- When $[\rho, H] = 0$, perform TPM
- When $[\rho, H] \neq 0$, perform the operator of work scheme

$\Downarrow$

Conditions (A), (A'), and (B) are satisfied simultaneously!

However,
the corresponding measurement is not continuous wrt ρ

The ultimate No-Go

Result 2

No measurement exists for which all $M_W(\rho)$ are continuous in ρ and the conditions (A), (A'), and (B) are satisfied simultaneously.

The ultimate No-Go

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No measurement exists for which all $M_W(\rho)$ are continuous in ρ and the conditions (A), (A'), and (B) are satisfied simultaneously.

Result 3

If none of the outcomes $\{W\}$ goes to 0 as $U \to \mathbb{I}$, then the scheme cannot simultaneously satisfy (A) and (A') — the two expressions of energy conservation.

Take-home messages

- For a linear measurement, satisfying Jarzynski's equality is equivalent to being the TPM scheme.
- Continuous nonlinear measurements cannot reconcile energy conservation and Jarzynski's equality.
- If a scheme has work outcomes that do not depend on the unitary evolution operator, then it cannot respect energy conservation.

arXiv:2104.09364